Marin Manouverability of ships

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Problem description

- Describe manouvring behaviour of (propelled) ships in terms of
  - Steering parameters
    - rudder angle, thrust
  - design parameters of the ship
    - Mass, length, moment of inertia, water displacement..

- Focus: stability behaviour for steady manouvres (equilibria).

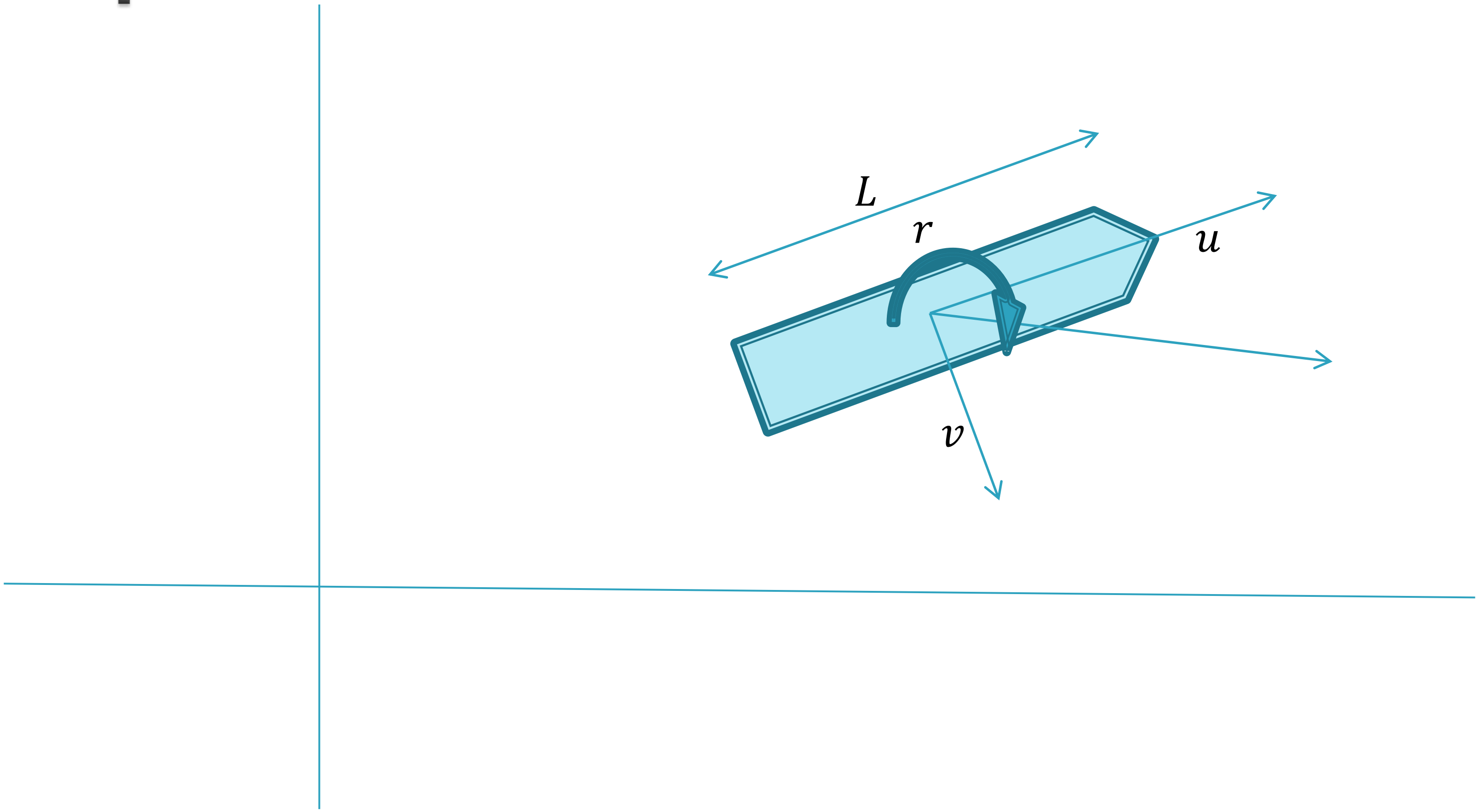
Model description

- Model of ship with forces...
  - Thrust of the propellor
    - possibly directed by rudder
  - Dissipative friction with the water
  - Coriolis force
    - due to choice of coordinate comoving with the ship

- Effective masses
  - Effect of dragging along the water by ship
  - Choice of coordinate system

- Lots and lots of coefficients:
  - functional dependence of forces on velocity

Ship vs earth coordinates



Model equations in rotating ship coordinates

- $M\,\dot{\boldsymbol{v}} = F = F^{\perp}(\boldsymbol{v}) + F_D(\boldsymbol{v},\alpha) + F_T(\boldsymbol{v},\alpha)$

- Where $\boldsymbol{v} = (u, v, Lr)$, and
- M symmetric mass matrix
- $F^{\perp} = (mrv, -mru, 0)$ Coriolis force $\perp \boldsymbol{v}$
- F_D is a dissipative friction force
- F_T propulsion force
- α steering angle

Dissipative force with rudder

$$X_{hull} = \frac{1}{2}\rho L_{pp} T \left(X'_{u'|u'|} u|u| + X'_{\beta\gamma} L_{pp} v\, r \right)$$

$$Y_{hull} = \frac{1}{2}\rho L_{pp} T \Big(Y'_{\beta}|u|v + Y'_{\gamma} L_{pp} u\, r + Y'_{\beta|\beta|} v|v| + Y'_{\gamma|\gamma|} L^2_{pp} r|r|$$
$$+ Y'_{\beta|\gamma|} L_{pp} v|r| + Y'_{|\beta|\gamma} L_{pp}|v|r + Y'_{ab}\left|u^{ay} v^{by}\right| \operatorname{sign}(v) V^{-ay-by+2} \Big)$$

$$N_{hull} = \frac{1}{2}\rho L^2_{pp} T \Big(N'_{\beta} u\, v + N'_{\gamma} L_{pp} r|u| + N'_{u'\gamma c} L^{c_n}_{pp} |u\, r^{c_n}| V^{-c_n+1} \operatorname{sign} r$$
$$+ N'_{r|r|} r|r| L^2_{pp} + N'_{\beta|\beta|} v|v| + N'_{\beta\beta\gamma} r\, v^2 L_{pp} V^{-1}$$
$$+ N'_{\beta\gamma\gamma} v\, r^2 L^2_{pp} V^{-1} \operatorname{sign} u + N'_{ab} |u^{a_n} v^{b_n}| V^{-a_n-b_n+2} \operatorname{sign}(u\, v) \Big)$$

$$X_R = -\frac{1}{2}\rho V^{-1}_{rr} A_R C_L \left(\frac{C_L u_r}{\pi\Lambda} (u_r \sin\delta - v_r \cos\delta)^2 + v_r (u_r \sin\delta - v_r \cos\delta)(u_r \cos\delta + v_r \sin\delta) \right)$$

$$Y_R = \frac{1}{2}(1 + a_H)\rho V^{-1}_{rr} A_R C_L \left(u_r (u_r \sin\delta - v_r \cos\delta)(u_r \cos\delta + v_r \sin\delta) - \frac{C_L v_r}{\pi\Lambda} (u_r \sin\delta - v_r \cos\delta)^2 \right)$$

$$N_R = Y_R\, x_r - X_R\, y_r$$

where

$$V_{rr} = \sqrt{u_r^2 + v_r^2}$$

$$u_r = u_p + C_{rue} \left(\sqrt{u_p^2 + \frac{8 T_p(u)}{\rho \pi D_p^2}} - u_p \right), \quad \text{case for ahead speed with positive thrust}$$

$$v_r = C_{db}\, v + C_{dr}\, x_r\, r$$

$$u_p = (1 - w) u$$

Thrust with rudder model

- $X_{prop} = T(u)$
- $Y_{prop} = 0$
- $N_{prop} = 0$

- In the example $T(u)$ decreases as $u \to \infty$ (?)
  - Power becomes less effective as you move faster through the water.

Model equations in rotating ship coordinates (with fixed thrust)

- $M\,\dot{\boldsymbol{v}} = F = F^{\perp}(\boldsymbol{v}) + F_H(\boldsymbol{v}) + F_T(\alpha)$

- Where $\boldsymbol{v} = (u, v, Lr)$, and
- M symmetric mass matrix
- $F^{\perp} = (mrv, -mru, 0)$ Coriolis force $\perp\ \boldsymbol{v}$
- F_H is a dissipative friction force from Hull
- F_T propulsion force
- α steering angle

Thruster model

- Only keep the Hull force
- Use fixed thrust in fixed direction to steer.
  - $X = \tau\cos(\alpha)$
  - $Y = \tau\sin(\alpha)$
  - $N = x_r\tau\sin(\alpha)$

  - E.g. take $x_r = -0.5L$

Equilibria

- Equilibrium is solution with $\dot{\boldsymbol{v}} = 0$ i.e.
- $F = 0$ we are looking for stationary points
  - in the *ships* coördinate system.
  - In *earth* coordinate system: ship moves on straight line or circle with angular velocity $\omega = r + \operatorname{arctg}\left(\frac{v}{u}\right)$ and radius $\frac{\sqrt{u^2+v^2}}{\omega}$

Proof:

- Earth coordinates $(x, y, \phi)$where
  - (x,y) is position of centre of gravity of the ship.
  - ϕ means the orientation angle of the ship

Write $w = u + i\, v,$ and $z = x + iy$, then

$$\dot{z} = w\, e^{i\phi}$$

$$\dot{\phi} = r + \arg(w) = \omega$$

Now we have $\phi = \omega t + \phi_0$ and

$$z = z_0 + \left(\frac{w e^{i\phi_0}}{i\omega}\right) e^{i\omega \mathrm{t}}$$

i.e. the radius of the circular motion is $\frac{|w|}{\omega}$ and angular velocity is ω.

Obvious equilibrium.

- Both the models have an "obvious" equilibrium at $\alpha = 0$: moving straight ahead
  - $u = constant(\tau)$
  - $v = 0$
  - $r = 0$
  -

How to study stability

- Linearise F in a stationary point.
- Stability determined by Jacobian $DF = \frac{\partial F}{\partial \boldsymbol{v}}$
  - A real 3×3 matrix
  - All depends on eigenvalues:
    - All negative → stability
    - 2 real negative, 1 real positive
      - Convergence on a plane, divergence in one direction
    - 1 real negative , 2 negative real part
      - spiralling in, converging to a plane
    - 1 real negative, 2 positve real part
      - Spiralling out converging to plane in one direction
    - ....

Result

- Stability depends on design parameters!
  - Not necessarily bad!

- Example: tune total mass (keep rest fixed)
  - $m' < m'^*$ 3 negative eigenvalue (stability)
  - $m' > m'^*$ 2 negative, 1 posive eigenvalue

- Explicit model → 1 Obvious negative eigenvalue, what are possibilities for all 3 ?

Fundamental assumption

- Friction force is dissipative, i.e.

$$\boldsymbol{v} \cdot F_D\,(\boldsymbol{v}, \alpha)\; < 0$$

- More precisely: for given thrust and α, if $|\boldsymbol{v}| \gg 0$ kinetic energy will decrease

$$\frac{dE}{dt} = \frac{1}{2}\frac{d}{dt}(\boldsymbol{v} \cdot \boldsymbol{M}\boldsymbol{v}) = \boldsymbol{M}\boldsymbol{v} = \;\; \boldsymbol{v} \cdot F^{\perp} + \boldsymbol{v} \cdot F_D(\boldsymbol{v}) + \boldsymbol{v} \cdot F_T < 0$$

Existence of a solution

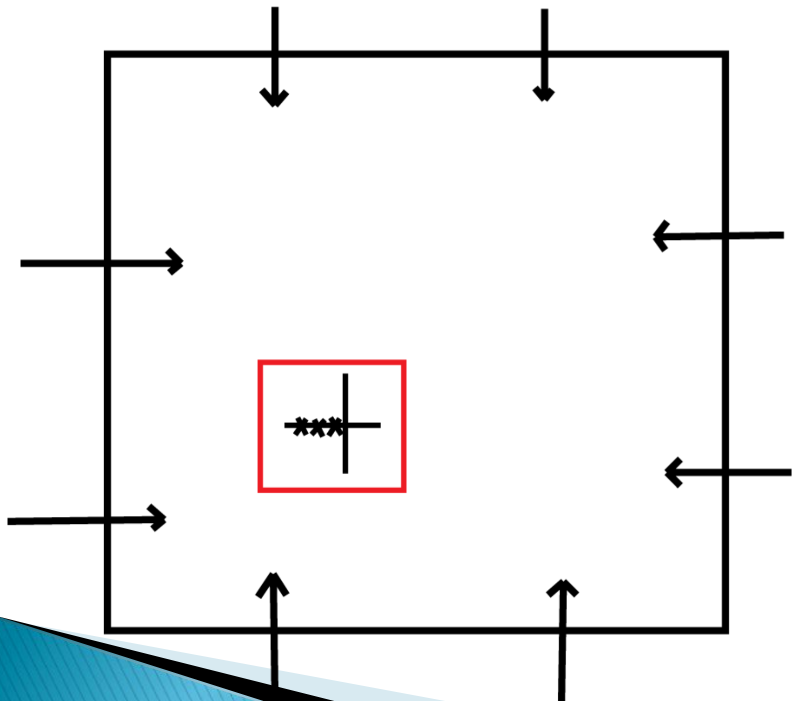
- From assumptions there *always* exists an equilibrium
  - There exist −1 solution counted with multiplicity.
  - Topological argument

Existence of a solution

- On a large $\boldsymbol{v}$ sphere $v \cdot F < 0$ (inward pointing)
- On the sphere, F is *homotopic* to the vectorfield $-\boldsymbol{v}$ as a map from $S^2 \to R^3 - 0$ i.e.
  - $(1-s)F + s(-\boldsymbol{v}) \neq 0$ for all $s \in [0,1]$.
  - This is obvious because $(1-s)F \cdot v + s(-v) \cdot v < 0$ for all $s \in [0,1]$.
- Therefore $\deg(F) = \deg(-id) =-1$
  - Homotopic maps have the same degree (so it is very robust)
  - To compute the degree of (-id) use the formula on the next slide.
  - The degree really is the map $\deg(F): \pi_2(S^2) = \boldsymbol{Z} \to \pi_2(\boldsymbol{R}^3 - 0) = \pi_2(S^2) = \boldsymbol{Z}$ induced on the homotopy groups, and is therefore an integer. It is the two dimensional analog of the winding number. There are many equivalent definitions for smooth maps F, for example, take any volume form μ on S^2 then $\deg(F) = \int_{S^2} F^* \mu \,/ \int_{S^2} \mu$. If you take a volume form concentrated near a generic point then you see the degree is the number of inverse images counted with orientation.

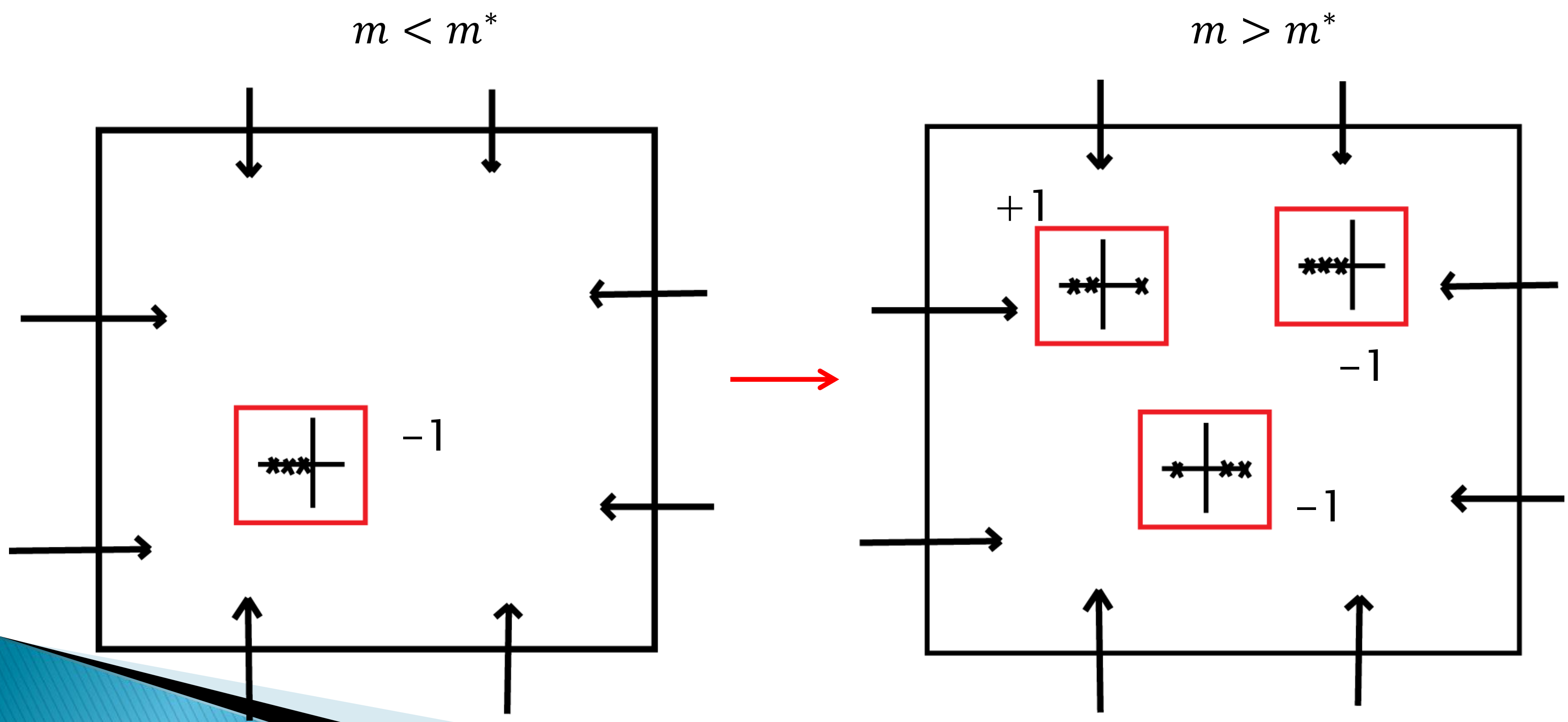
−1 solutions

$$\deg(F) = \sum_{F(p)=0} sign(\det(DF(p)))$$

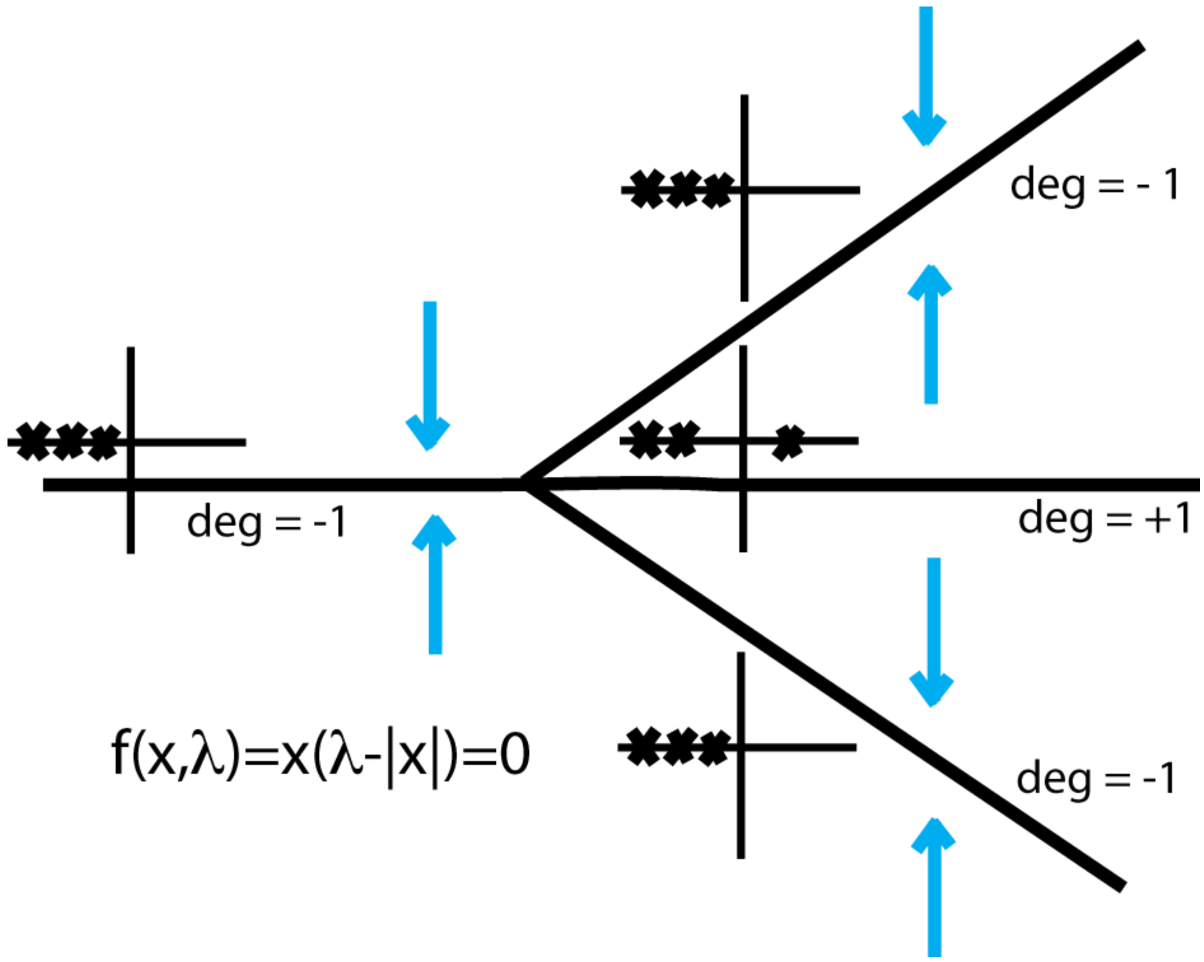


When 1 eigenvalue becomes zero

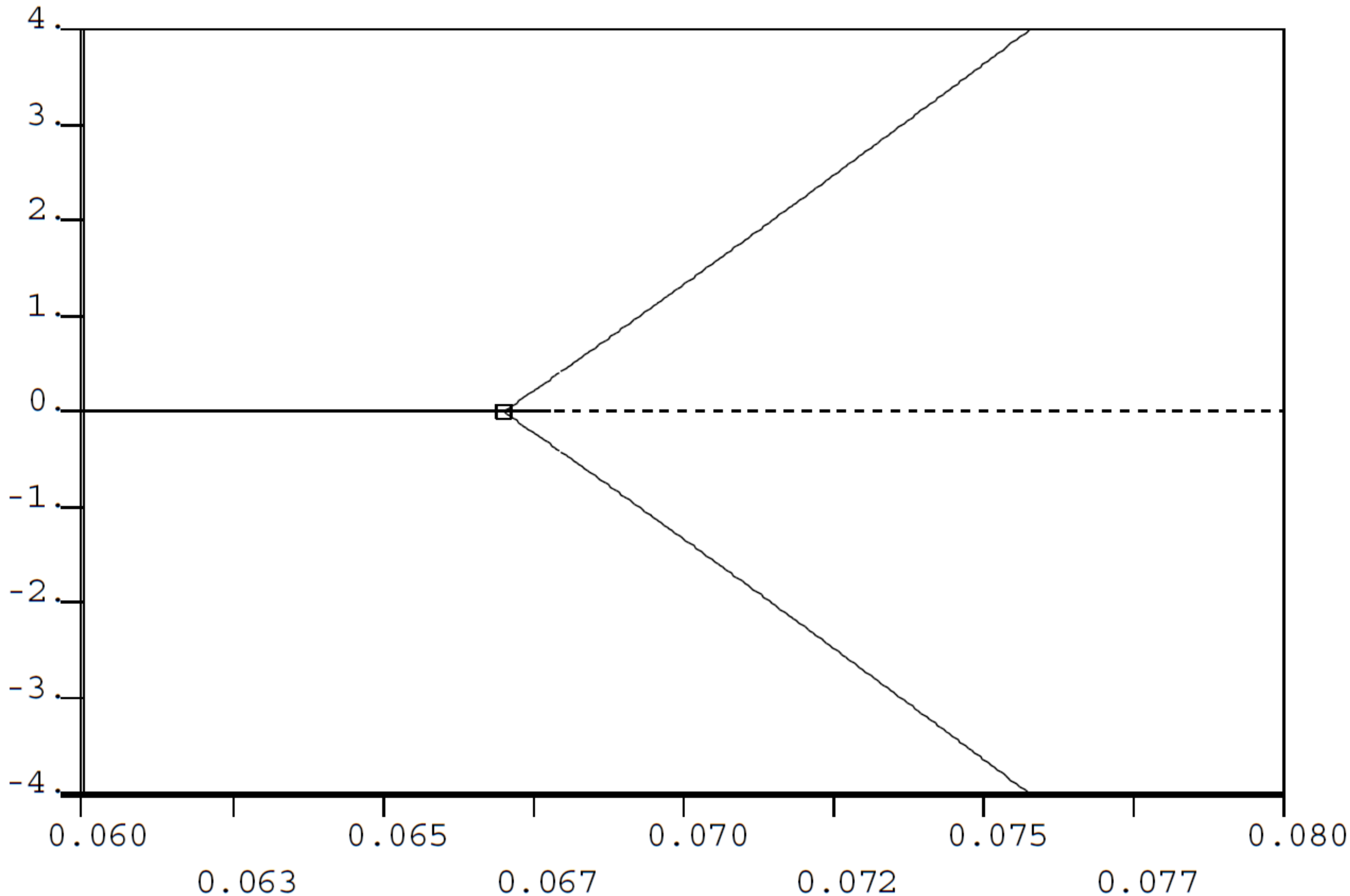
$$\deg(F) = \sum_{F(p)=0} sign(\det(DF(p)))$$



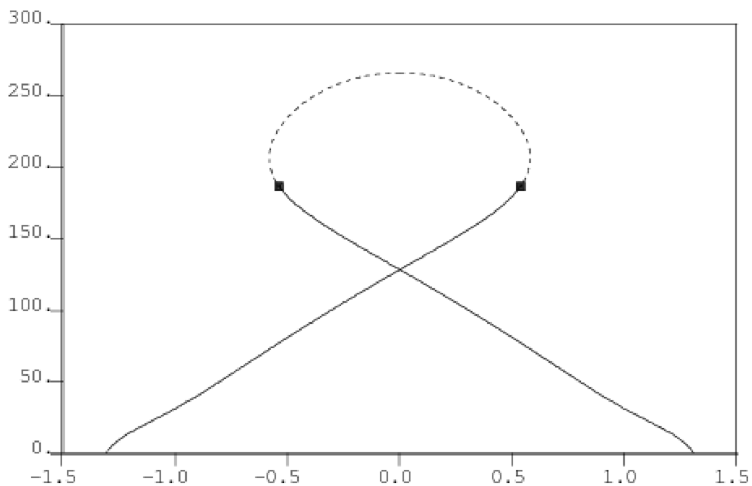
Bifurcation diagram



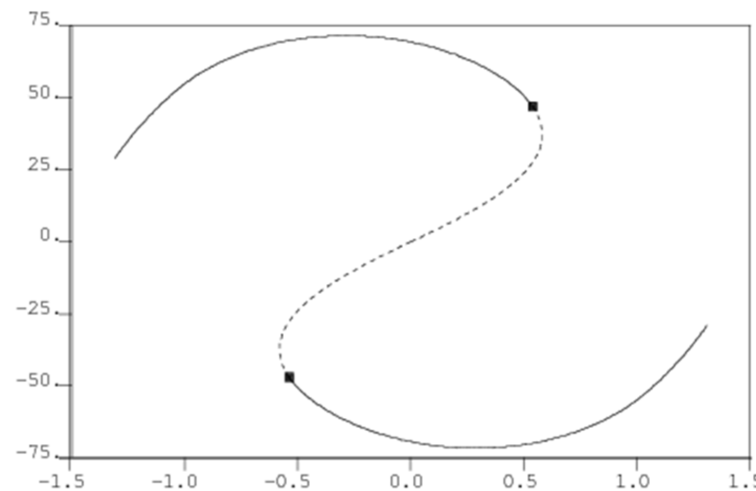
What you find numerically



tuning α : Bifurcation diagrams of equilibria:

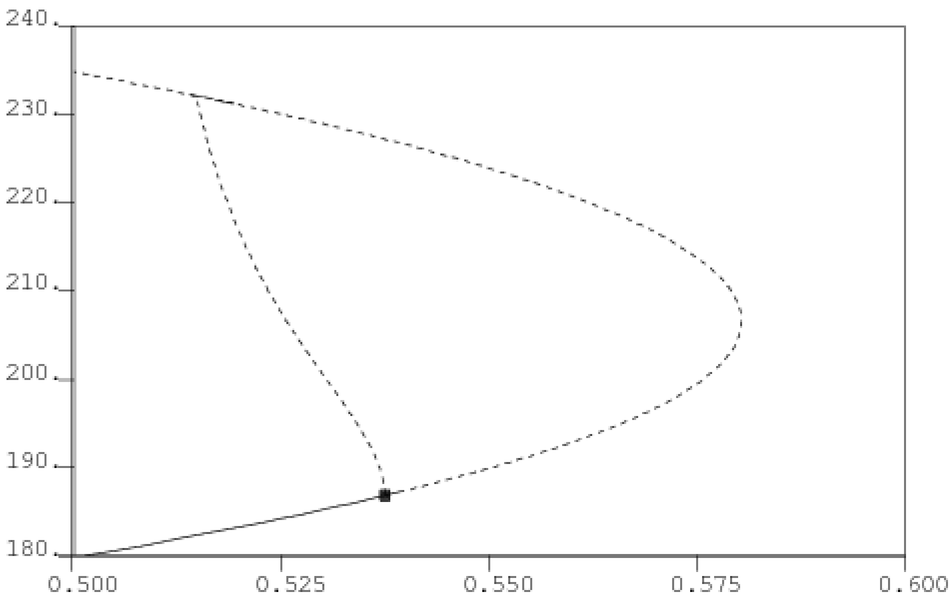


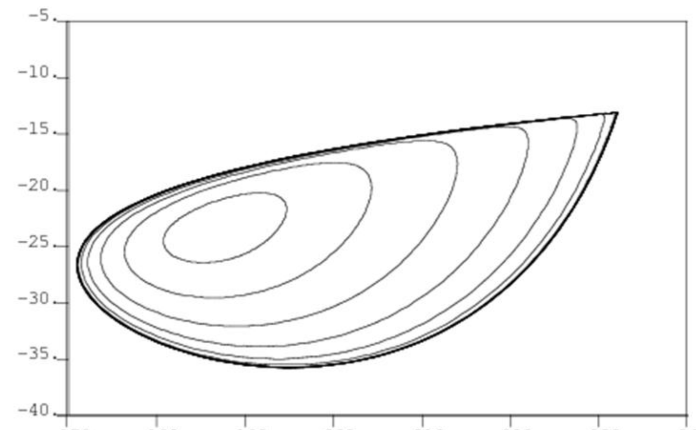
u vs. α

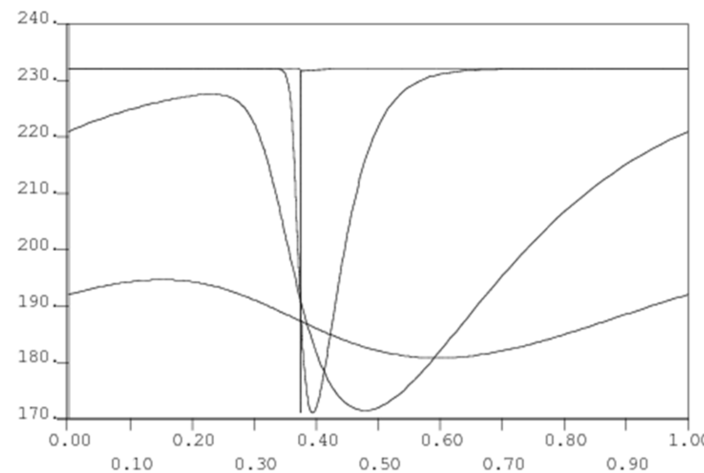


r vs. α

Bifurcation of periodic orbits

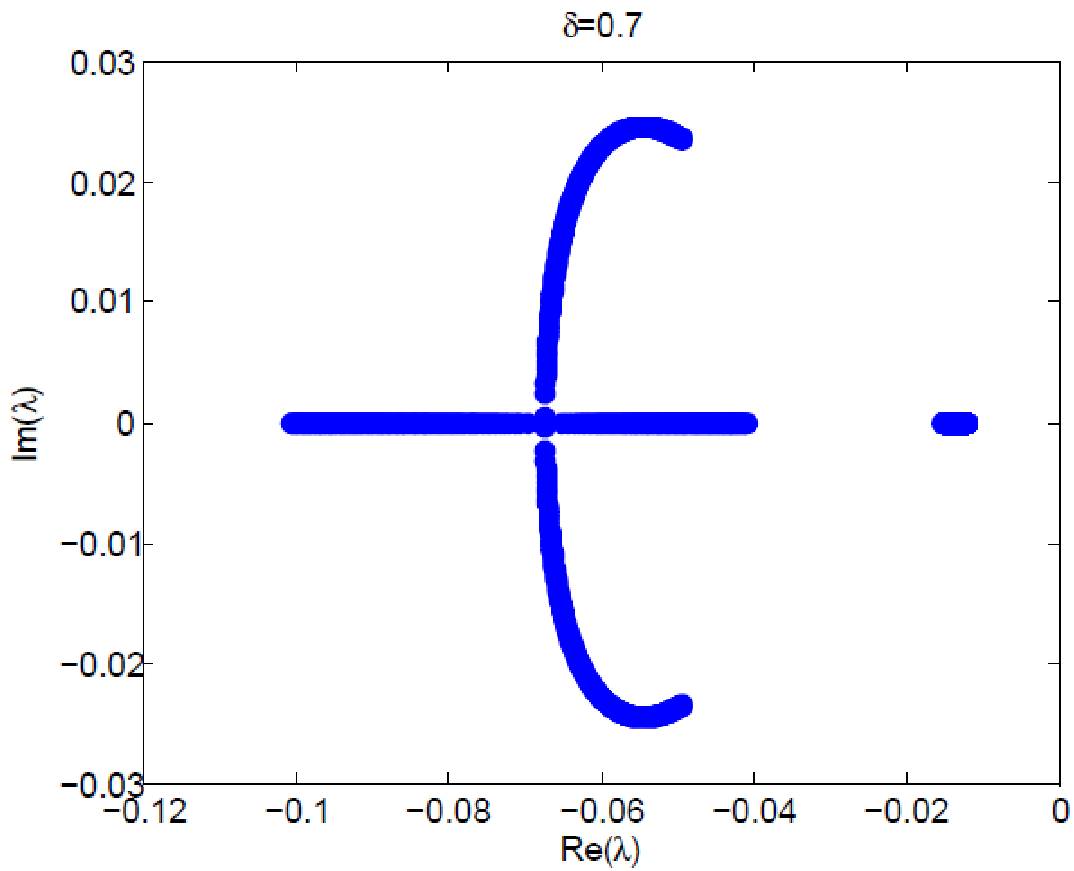




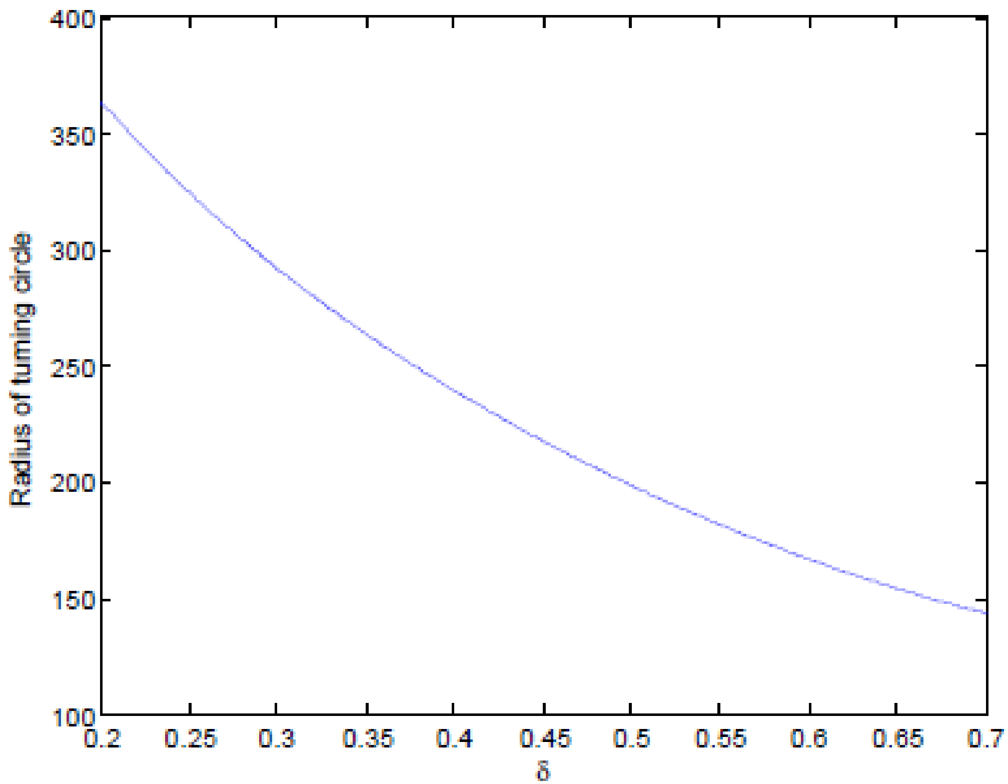


- Rudder Model

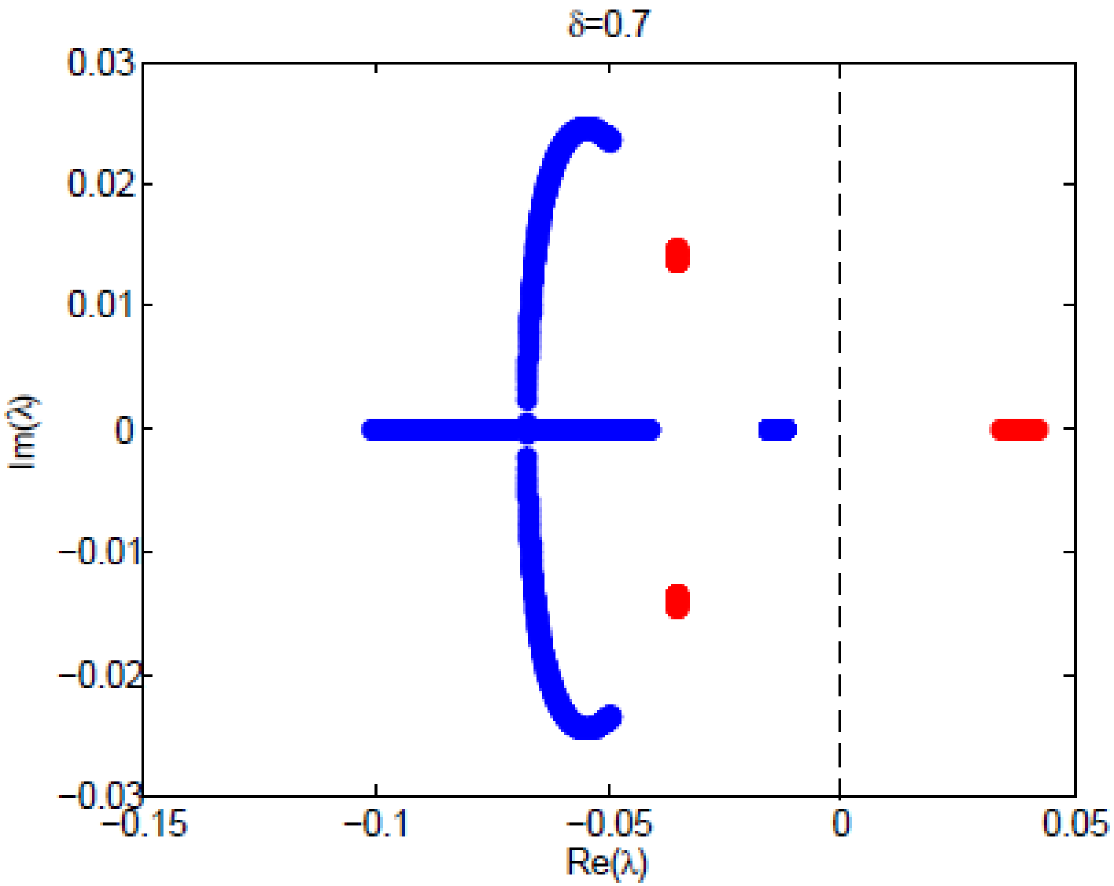
Eigenvalue trajectories for the rudder model



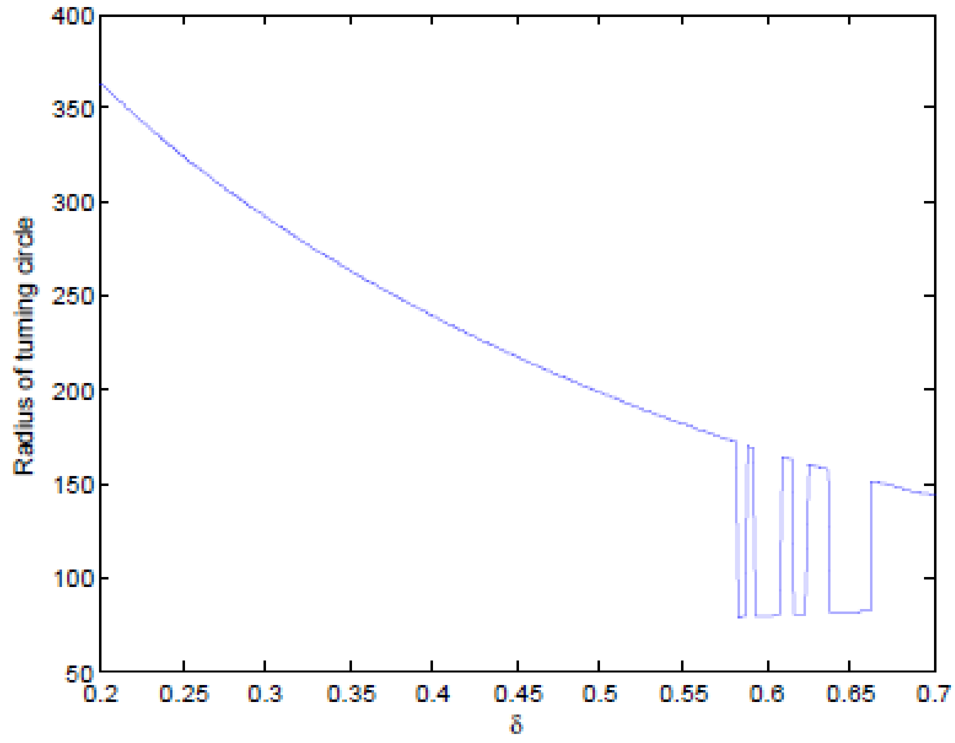
Turning circle vs Rudder angle



Eigenvalue trajectories



Turning circle vs Rudder angle



Outlook

- Analysis of stationary points and dynamical system point of view, is a useful toolbox.
- Specialised numerical methods for following stationary points is (fast) alternative to solving full equations.
- Open question: stabilisation by control.

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